NUMERICAL ANALYSIS
Assignment -9 (week 9)
Total Marks - 25
Posted on - 18/9/2017 (Monday);
To be submitted on or before-27/9/2017 (Wednesday), 23.59 hours.

Problems on

- Fixed Point Iteration

INSTRUCTIONS

- This is a question paper cum answer booklet.
- Take a print out of this.
- Present the details of the computations of the solution of each problem **which you will have to show** in the space provided at the bottom of the page.
- Fill in the answers in the space provided below each question.
- Scan the booklet and submit it as a pdf file before the deadline for evaluation.

1. A sequence x_n is defined by (a) $x_0 = 5$, $x_{n+1} = \frac{x_n^4}{16} - \frac{x_n^3}{2} + 8x_n - 12$.
Show that x_n has cubic convergence to $p = 4$.
Fill in the blanks:
(i) $g(4) =$ ____________; (ii) $g'(4) =$ ____________; (iii) $g''(4) =$ ____________;
(iv) If $e_{n+1} = Ce_n^\alpha$, then $\alpha =$ ____________ .
Here x_n generates successive iterates for a fixed point of the function g using $x_{n+1} = g(x_n)$.
(b) Calculate the smallest integer 'n' for which the inequality $|x_n - p| < 10^{-6}$ is valid.
Fill in the blanks:
(i) If $e_{n+1} = Ce_n^3 + O(e_n^4)$, then $C =$ ____________
(ii) $e_n = C^{\beta(n)}$, then $\beta(n) =$ ____________
(iii) $n \geq$ ____________ . (5+5=10 marks)

Show your work for the solution of problem 1 in the space provided below.

2. The following methods are proposed to compute $(21)^{\frac{1}{3}}$. Determine the order of convergence of each of these methods.
(a)$x_n = \frac{1}{21}[20x_{n-1} + \frac{21}{x_{n-1}^2}]$.
(b) $x_n = x_{n-1} - \frac{x_{n-1}^3 - 21}{3x_{n-1}^2}$.
Fill in the blanks:
(i) order of convergence of (a) is ______________,

(ii) order of convergence of (b) is ______________ . (3+3=6 marks)

Show your work for the solution of problem 2 in the space provided below.

3. The error in an iteration method $x_{n+1} = g(x_n)$ is known to decrease geometrically. Show that, from three consecutive iterates x_{n-1}, x_n, x_{n+1}, a new approximation to the exact root p may be given by
$p = \frac{x_{n+1}x_{n-1} - x_n^2}{x_{n+1} - 2x_n + x_{n-1}}$
Fill in the blanks:
(a) If the error at the n^{th} step is denoted by e_{n-1}, then
(i) error at n^{th} step is = ________________
(ii) error at $(n+1)^{th}$ step is = ________________
(iii) $x_{n+1}x_{n-1} - x_n^2 =$ ________________.
(iv) $x_{n+1} - 2x_n + x_{n-1} =$ ________________ (4 marks)

Show your work for the solution of problem 3 in the space provided below.

4. We have shown in one of the practice problems that $g(x) = 2^{-x}$ has a unique fixed point on $[\frac{1}{3}, 1]$. Use fixed point iteration method to find an approximation to the fixed point that is accurate to within 10^{-4}.
(Hint: start with $p_0 = 1$ and perform 12 iterations)
Examine the number of iterations required to achieve 10^{-4} accuracy. (Hint: We show in practice problem that $k = 0.551$. Use this in your solution).
Fill in the blanks: (5 marks)
(a) $n \simeq$ ______________; (b) $p_1 =$ ______________ (c) $p_{12} =$ ______________ .

Show your work for the solution of problem 4 in the space provided below.